



**K R E A T R Y X**

**K** Notes

┌ **ELECTRICAL MACHINES** ┐

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## Manual for K-Notes

### Why K-Notes?

Towards the end of preparation, a student has lost the time to revise all the chapters from his / her class notes / standard text books. This is the reason why K-Notes is specifically intended for Quick Revision and should not be considered as comprehensive study material.

### What are K-Notes?

A 40 page or less notebook for each subject which contains all concepts covered in GATE Curriculum in a concise manner to aid a student in final stages of his/her preparation. It is highly useful for both the students as well as working professionals who are preparing for GATE as it comes handy while traveling long distances.

### When do I start using K-Notes?

It is highly recommended to use K-Notes in the last 2 months before GATE Exam (November end onwards).

### How do I use K-Notes?

Once you finish the entire K-Notes for a particular subject, you should practice the respective Subject Test / Mixed Question Bag containing questions from all the Chapters to make best use of it.

## Transformers

### Impact of dimensions on various parameters of Transformer

KVA Rating  $\propto$  (Core Dimension)<sup>4</sup>

Voltage Rating  $\propto$  (Core Dimension)<sup>2</sup>

Current Rating  $\propto$  (Core Dimension)<sup>2</sup>

No-Load Current  $\propto$  Core Dimension

Core Loss  $\propto$  Core Volume

### Induced EMF in a Transformer

$$E_1 = N_1 \frac{d\phi}{dt}$$

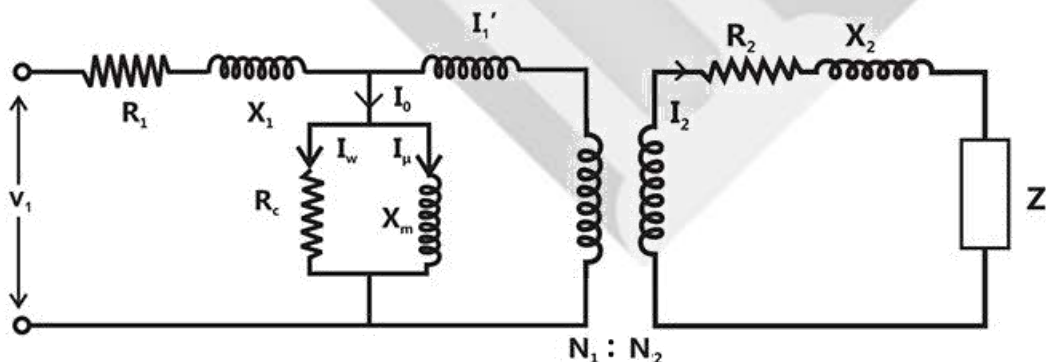
$$E_2 = N_2 \frac{d\phi}{dt}$$

$$E_1(\text{rms}) = 4.44fN_1\phi_m$$

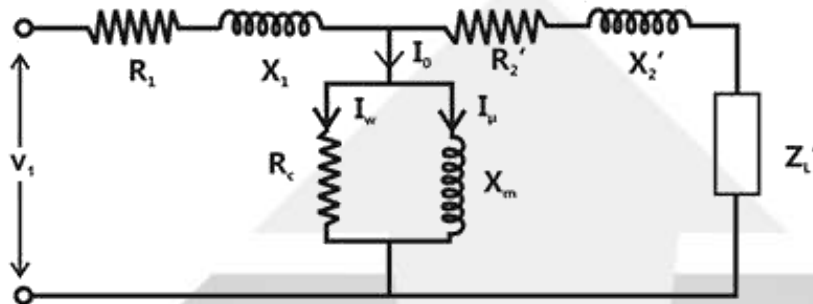
$$E_2(\text{rms}) = 4.44fN_2\phi_m$$

- Where  $E_1$  and  $E_2$  are emf in primary and secondary windings of Transformer respectively.
- $\Phi$  is the flux in the transformer and  $\Phi_m$  is maximum value of flux.
- The polarity of emf is decided on basis of Lenz Law as currents in primary and secondary should be such that primary and secondary flux should oppose each other.
- Also, primary current enters the positive terminal of primary winding as primary absorbs power and secondary current leaves the positive terminal of secondary winding as secondary delivers power and this way we can mark emf polarities.

### Exact equivalent circuit

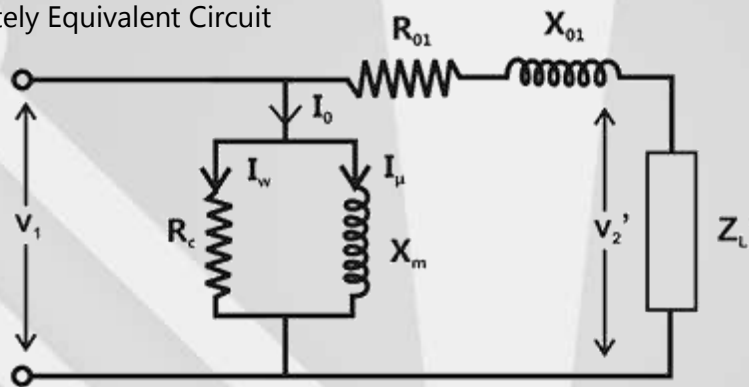


Exact equivalent circuit w.r.t. primary



$$R_2' = R_2 \left( \frac{N_1}{N_2} \right)^2 ; X_2' = X_2 \left( \frac{N_1}{N_2} \right)^2 ; Z_L' = Z_L \left( \frac{N_1}{N_2} \right)^2 ;$$

- Approximately Equivalent Circuit



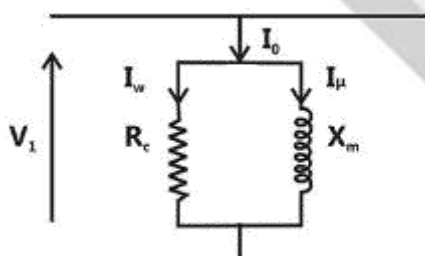
$$R_{01} = R_1 + R_2'$$

$$X_{01} = X_1 + X_2'$$

### Tests Conducted on a Transformer

#### (i) Open Circuit Test

- Conducted on LV side keeping HV side open circuited
- Equivalent Circuit

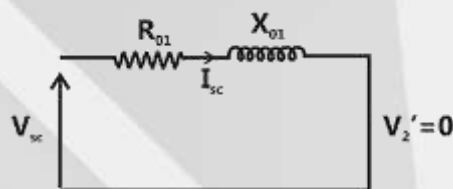


- Power reading =  $P = V_1 I_0 \cos \phi_0 = \frac{V_1^2}{R_c}$  ----- (i)
- Ammeter reading  $\Rightarrow I = I_0$
- $\cos \phi_0 = \frac{P}{V_1 I_0}$
- Calculate  $\sin \phi_0 = \sqrt{1 - \cos^2 \phi_0}$
- $Q = V_1 I_0 \sin \phi_0 = \frac{V_1^2}{X_m}$  ----- (ii)

Calculate  $R_c$  from (i) &  $X_m$  from (ii)

### (ii) Short Circuit Test

- Conducted on HV side keeping LV side short circuited
- Equivalent Circuit



- $R_{01}$  &  $X_{01}$  are equivalent winding resistance & equivalent leakage reactor referred to HV side.
- Wattmeter reading =  $P = I_{sc}^2 R_{01}$  from this equation, we can calculate  $R_{01}$
- $Z_{01} = \frac{V_{sc}}{I_{sc}}$  &  $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$
- We obtain  $R_{01}$ ,  $X_{01}$  & full load copper losses from this test.

### Losses on Transformers

- **Copper Loss**

$$P_{Cu} = I_1^2 R_1 + I_2^2 R_2$$

$$= I_1^2 R_{01} = I_2^2 R_{02}$$

Where  $I_1$  = primary current

$I_2$  = secondary current



$R_1$  = primary winding resistance

$R_2$  = secondary winding resistance

$$R_{01} = R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2 ; R_{02} = R_2 + \left(\frac{N_2}{N_1}\right)^2 R_1$$

○ **Core Loss**

**(i) Hysteresis Loss**

$$P_n = K_n B_m^x f$$

$$x = 1.6$$

$B_m$  = maximum value of flux density

$$P_n = K_n B_m^{1.6} f$$

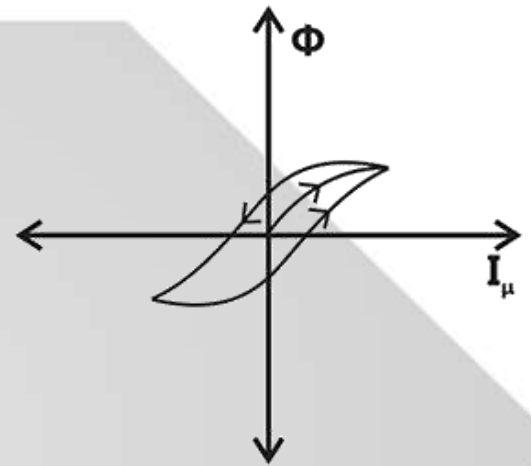
$$B_m \propto \frac{V}{f}$$

$V$  = applied voltage

$f$  = frequency

$$P_n = K_h' \left(\frac{V}{f}\right)^{1.6} f = K_h' V^{1.6} f^{-0.6}$$

If  $V$  is constant &  $f$  is increased,  $P_h$  decreases



**(ii) Eddy Current Loss**

$$P_e = K_e B_m^2 f^2$$

$$B_m \propto \frac{V}{f}$$

$$P_e = K_e' \left(\frac{V}{f}\right)^2 f^2 = K_e' V^2$$

$$\text{Core loss} = P_c = P_e + P_n$$

## Efficiency

$$\eta = \frac{x(\text{KVA})\cos\phi}{x(\text{KVA})\cos\phi + P_i + x^2 P_{\text{Cu,FL}}}$$

$X$  = % loading of Transformer

$\cos\phi$  = power factor

$P_i$  = iron loss

$P_{\text{Cu,FL}}$  = Full load copper losses

KVA = Power rating of Transformer

For maximum efficiency,

$$x = \sqrt{\frac{P_i}{P_{\text{Cu,FL}}}}$$

## Voltage Regulation of Transformer

$$\text{Regulation down} = \frac{V_{\text{NL}} - V_{\text{FL}}}{V_{\text{NL}}} \times 100$$

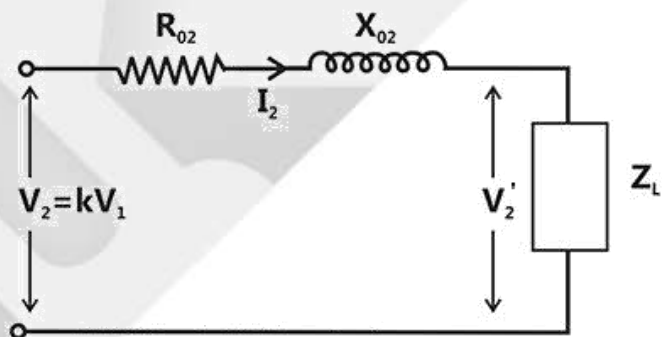
$$\text{Regulation up} = \frac{V_{\text{NL}} - V_{\text{FL}}}{V_{\text{FL}}} \times 100$$

Equivalent circuit with respect to secondary

$$K = \text{Transformation Ratio} = \frac{N_2}{N_1}$$

No-load voltage =  $V_2$

Full-load voltage =  $V_2'$



Approximate Voltage Regulation

$$\text{VR} = \frac{I_2 (R_{02} \cos\phi_2 \pm X_{02} \sin\phi_2)}{V_2}$$



$\cos\phi_2$  = power factor of load  $Z_L$

+ sign is used for lagging pf load

- sign is used for leading pf load

### Condition for zero voltage regulation

$$\phi_2 = \tan^{-1} \left( \frac{R_{02}}{X_{02}} \right)$$

The power factor is leading, Voltage Regulation can never be zero for lagging pf load.

### Condition for maximum voltage regulation

$$\phi_2 = \tan^{-1} \left( \frac{X_{02}}{R_{02}} \right)$$

The power factor is leading, Voltage Regulation can never be negative for lagging pf loads

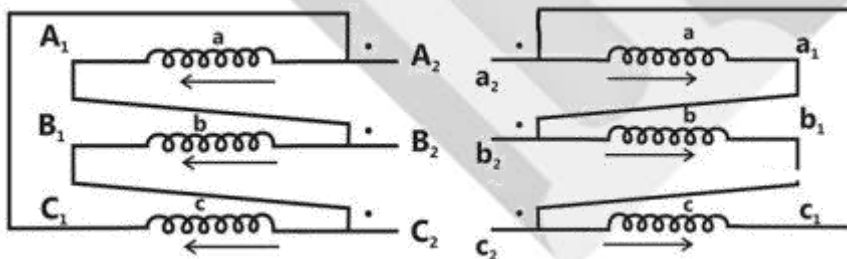
## Three – Phase Transformers

In a 3-Phase transformers; the windings placed parallel to each other at as primary & secondary of single phase transformer.

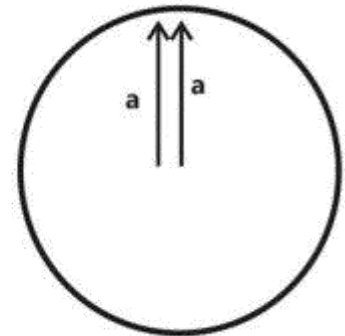
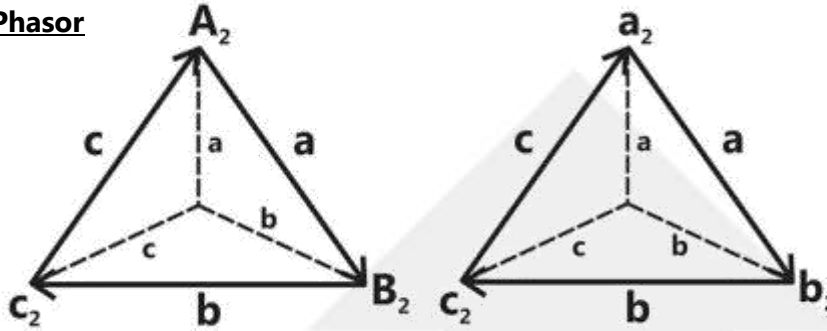
### Rules to draw Phasor diagram

- 1) Always draw phasors from A to B, B to C & C to A for line voltages.
- 2) The end points should have same naming as the input or output terminals.
- 3) If we draw primary phasor from dotted to undotted terminal and if secondary voltage is also from dotted to undotted, then secondary voltage is in same phase else in opposite phase.

Some examples



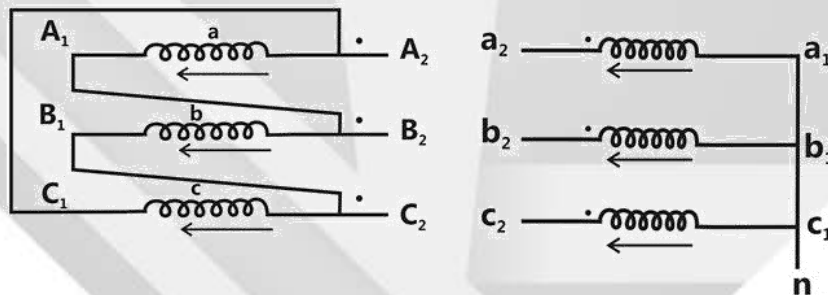
### Phasor



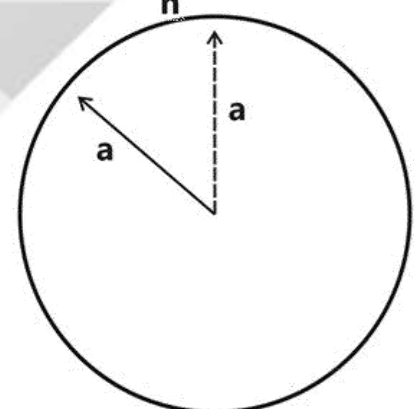
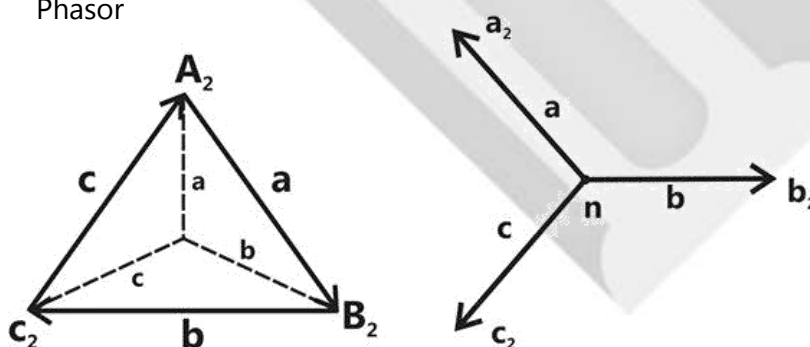
Dd 12 connection

- If you observe carefully, we traverse from dotted to undotted terminal in primary while going from  $a_2$  to  $b_2$ ,  $b_2$  to  $c_2$  &  $c_2$  to  $a_2$ .  
Same is the case when we traverse the secondary winding, so secondary voltage are in-phase to primary.
- Then, we draw reference phasors from neutral to terminal and mark it with phase with same name as terminal it is pointed to.  
Then we plot it on clock & we observe it is like 12 O' clock so name is Dd12 connection.

### Another example



### Phasor



Dy 11 connection

- Here, we traversed primary from dotted to undotted terminal & in secondary from undotted to dotted so all secondary phasor are out of phase wrt primary.

### Parallel operation of Transformer

#### Necessary Conditions

- 1) Voltage ratings of both transformers should be same.
- 2) Transformers should have same polarity.
- 3) Phase sequence of both transformers must be same in case of 3- phase transformers.
- 4) Phase displacement between secondary's of both transformers must be  $0^\circ$ .

If there are 2 transformers A & B supplying a load power  $S_L$ .

$$S_A = S_L \frac{Z_B}{Z_A + Z_B} ; S_B = S_L \frac{Z_A}{Z_A + Z_B}$$

$Z_B$  = impedance of transformer B (in ohms)

$Z_A$  = impedance of transformer A (in ohms)

### Auto Transformer

- Generally, auto transformer is created from 2- winding transformer.
- If rating of auto – transformer is LV/HV or HV/LV

LV = low voltage

HV = high voltage

$$\text{Transformation Ratio} = K = \frac{LV}{HV}$$

- KVA rating of auto transfer =  $\left( \frac{1}{1 - K} \right)$  (KVA rating of 2- winding Transformer)
- In auto- transformer, power is transferred from primary to secondary by 2 methods induction & conduction.
- $KVA_{\text{induction}} = (1 - K)(\text{Input KVA})$
- $KVA_{\text{conduction}} = K(\text{Input KVA})$
- % Full load losses =  $(1 - K)[\%FL \text{ losses in 2 – winding Transformer}]$
- If copper & core losses are not given separately, then we consider losses as constant, same as that of two winding transformer while calculating efficiency

## DC Machines

### Induced emf equation

$$E_a = \frac{\phi NZ}{60} \left( \frac{P}{A} \right)$$

$\phi$  = flux per pole(wb)

N = speed of machine(rpm)

P = number of poles

A = number of paralld path

Z = number of conductors

A = 2 for wave winding

A = P for lap winding

If speed is given in rad/sec

$$E_a = \frac{\phi \omega Z}{2\pi} \left( \frac{P}{A} \right) \quad \text{where } \omega = \text{speed (rad/s)}$$

$$= \left( \frac{PZ}{2\pi A} \right) (\phi \omega) = K_m \phi \omega$$

$$K_m = \frac{PZ}{2\pi A} = \text{machine constant}$$

### Developed Torque

$$T = K_m \phi I_a$$

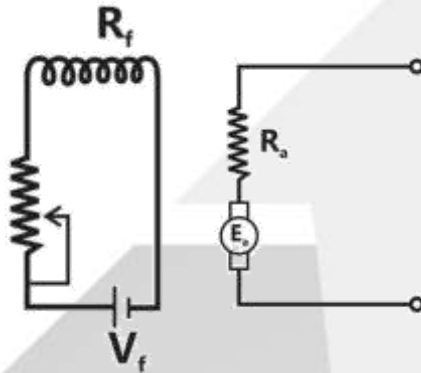
$$K_m = \frac{PZ}{2\pi A} = \text{machine constant}$$

$\phi$  = flux per pole

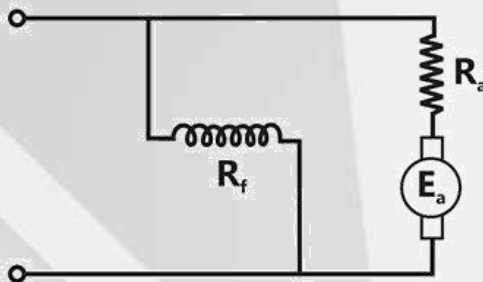
$I_a$  = armature current

## Classification of DC Machine

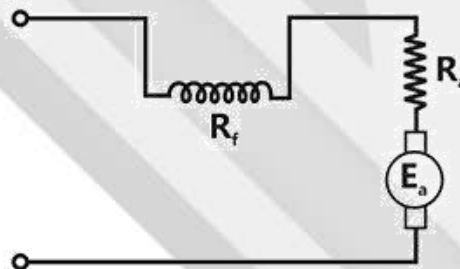
(i) Separately excited



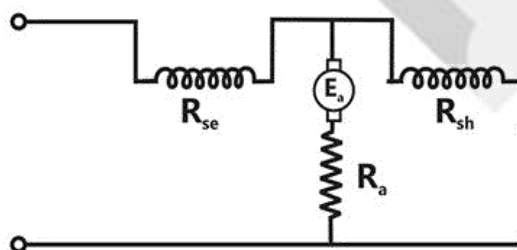
(ii) Shunt excited



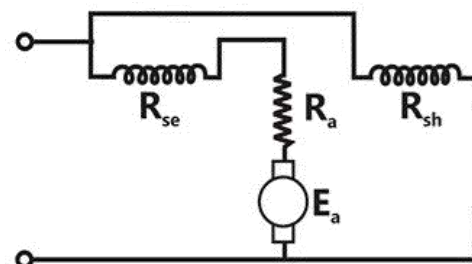
(iii) Series excited



(iv) Compound Excited



Short shunt



Long shunt

### Terminologies

$R_a$  : Armature Resistance

$R_{se}$  : Series Field winding Resistance

$R_{sh}$  : Shunt Field winding Resistance

- The only difference between Generator & Motor will be that the direction of armature current is coming out of positive terminal of emf  $E_a$ . In case of motor, armature current flows into  $E_a$ .

### Performance Equations of DC Machines

For shunt & separately excited machine

Generator:  $E_a = V_t + I_a R_a$

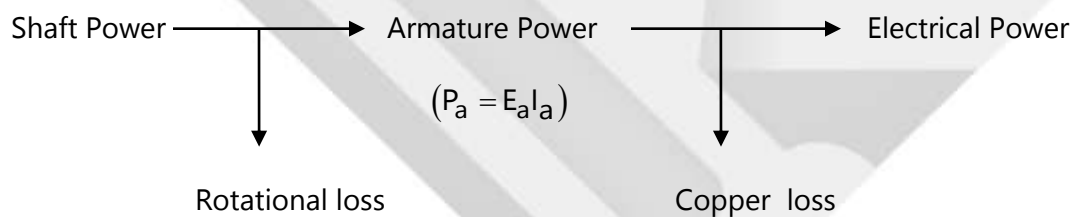
Motor:  $E_a = V_t - I_a R_a$

For series & compound excited machine

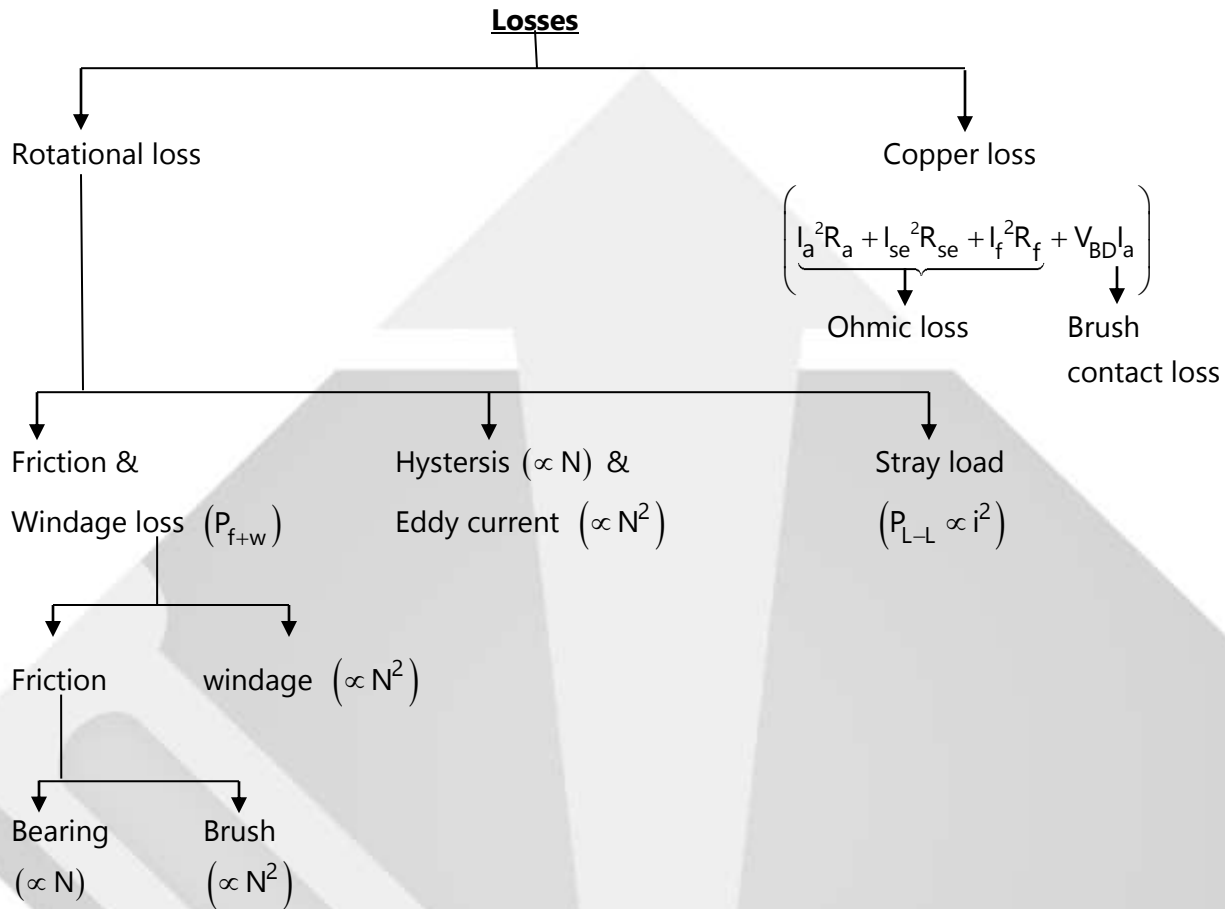
Generator:  $E_a = V_t + I_a (R_a + R_{se})$

Motor:  $E_a = V_t - I_a (R_a + R_{se})$

### Power Flow



- This power flow diagram is for a dc generator.
- If you traverse the diagram from right to left then it is a power flow diagram for a motor.



### Efficiency

$$\eta = \frac{V_a I_a}{V_a I_a + I_a^2 R_a + V_{BD} I_a + P_k}; \text{ for generator}$$

$P_k$  = sum of all constant loss

### For maximum efficiency

For shunt & separately excited machine  $I_a = \sqrt{\frac{P_k}{r_a}}$

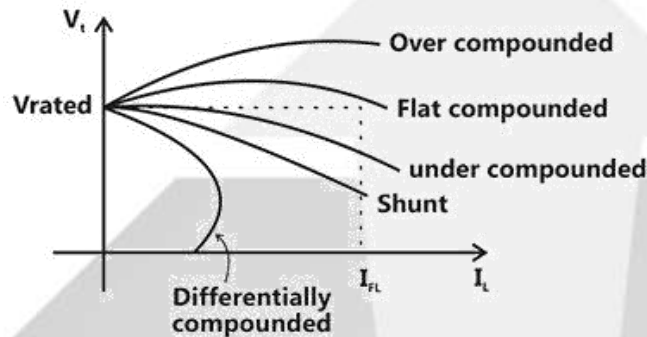
For series & compound excited machine  $I_a = \sqrt{\frac{P_k}{r_a + r_{se}}}$



## Characteristics of DC Generator

### External characteristics

If no-load voltage is same for all types of generators:

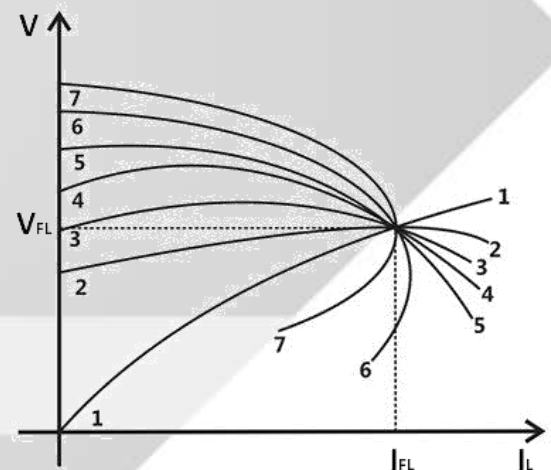


There are two categories of compound generators/motors

1. Cumulative Compound => If series field flux aids the shunt fields flux.
2. Differentially Compound => If series field flux opposes the shunt field flux.

If full – load voltage of all generators is kept same

- |                    |                             |
|--------------------|-----------------------------|
| 1 → series excited | 5 → separately excited      |
| 2 → over compound  | 6 → shunt excited           |
| 3 → level compound | 7 → differentially compound |
| 4 → under compound |                             |



Conditions for voltage build-up in Shunt Generator

- 1) There must be residual flux.
- 2) Correct polarity of field winding with respect to armature winding so that field flux aids residual flux for a given direction of rotation.
- 3) Field Resistance must be less than critical value

$$R_f < R_{f(cr)}$$

Critical resistance is equal to the slope of air-gap line.

- 4) Speed of rotation should be more than critical value for a given field resistance  $R_f$ .

$$N > N_{cr}$$

## Braking of DC Motor

### Plugging

- Supply to armature terminals is reversed while field is left undisturbed.
- The current reverses resulting into negative torque & that brings rotor quickly to rest.

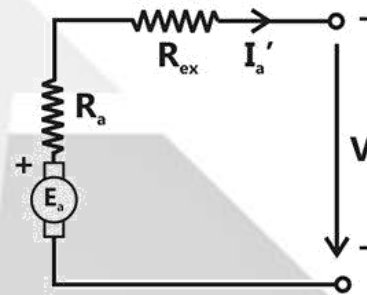
$$I'_a = \frac{(V + E_a)}{(R_a + R_{ex})}$$

- Plugging Torque =  $\frac{E_a I'_a}{\omega}$ ,  $\omega$  = speed of rotor

$$\text{Before plugging, } I_a = \frac{(V - E_a)}{R_a}$$

$$\text{Load Torque} = \frac{E_a I_a}{\omega}$$

$$\text{Braking Torque} = (\text{Load Torque} + \text{Plugging Torque})$$



## Synchronous Machine

### Induced emf

$$\text{Phase voltage} = 4.44 N_{ph} \phi f$$

$N_{ph}$ : number of turns per phase

$\phi$ : flux per pole

$f$ : frequency

This phase voltage is rms value

### Armature Winding

- Usually, coil span is  $180^\circ$  (electrical)
- If coil span =  $180^\circ$  (electrical), coil is called as full pitch coil.
- If coil span =  $(180^\circ - \epsilon)$  (electrical), coil is called as Chorded coil or short pitched winding.

- Pitch Factor,  $K_p = \cos \frac{\epsilon}{2}$
- Induced emf =  $(4.44 N_{ph} \phi f K_p)$
- For  $n^{\text{th}}$  harmonic  
Induced emf =  $(4.44 N_{ph} \phi f K_p)$

$$K_p = \cos \left( \frac{n \epsilon}{2} \right)$$

To eliminate  $n^{\text{th}}$  harmonic

$$\frac{n \epsilon}{2} = \frac{\pi}{2}$$

$$\epsilon = \frac{180^\circ}{n} \text{ (electrical)}$$

### **Distributed Winding**

$$m = \frac{\text{number of slots}}{\text{number of poles} \times \text{no. of phase}}$$

$$\text{Coil Span} = \frac{\text{number of slots}}{\text{number of poles}}$$

$$\beta = \frac{180^\circ}{\text{coil span}} \text{ (electrical);}$$

$$\text{Distribution Factor, } K_d = \frac{\sin \left( \frac{m\beta}{2} \right)}{m \sin \left( \frac{\beta}{2} \right)}$$

For  $n^{\text{th}}$  harmonic,  $\beta$  is replaced by  $n \beta$

$$K_d = \frac{\sin \left( \frac{mn\beta}{2} \right)}{m \sin \left( \frac{n\beta}{2} \right)}$$

- For uniform distribution replace  $\sin\left(\frac{n\beta}{2}\right)$  by  $\frac{n\beta}{2}$

Winding Factor,  $K_w = K_p K_d$

Induced emf  $= 4.44 \phi N_{ph} f K_w$

### Armature Resistance

Generally winding resistance is measured using voltmeter ammeter –method.

For star connection

$$R_m = \frac{V}{I} = \frac{\text{voltmeter reading}}{\text{ammeter reading}}$$

$$R_m = 2R$$

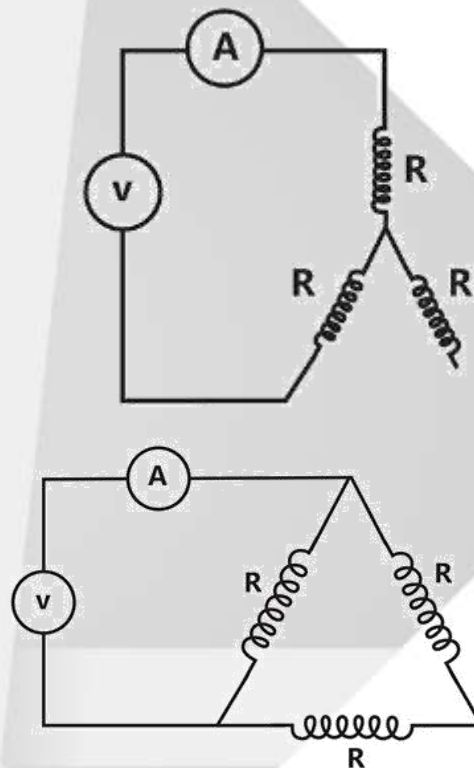
$$R = \frac{R_m}{2}$$

For Delta Connection

$$R_m = \frac{\text{voltmeter reading}}{\text{ammeter reading}}$$

$$R_m = \frac{2}{3}R$$

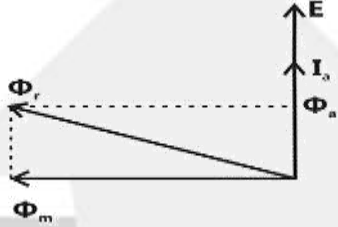
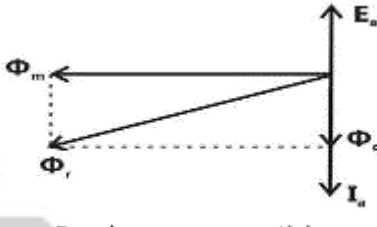
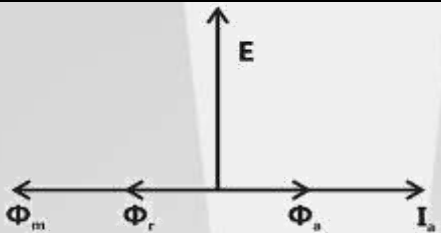
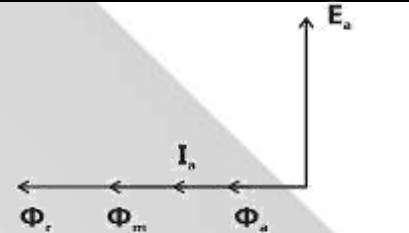
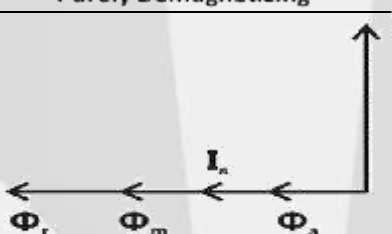
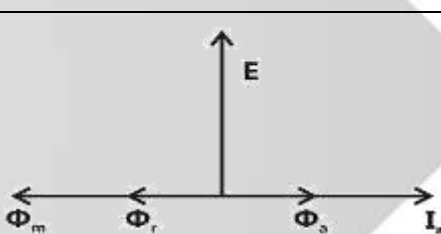
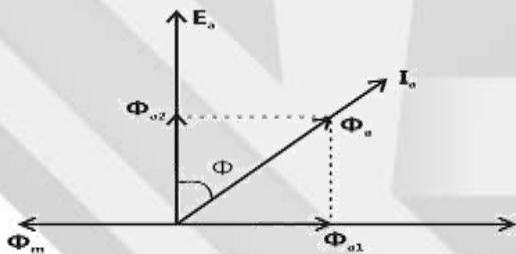
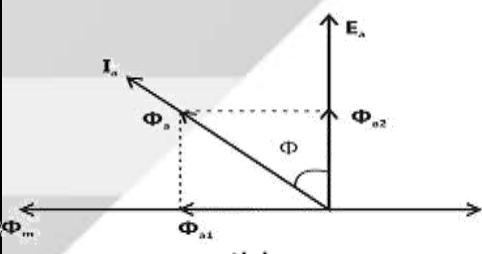
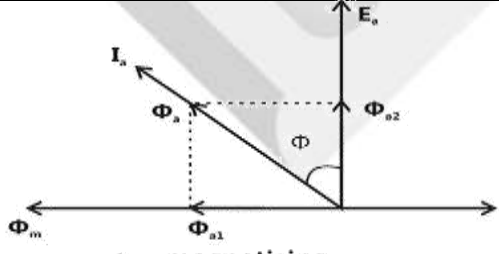
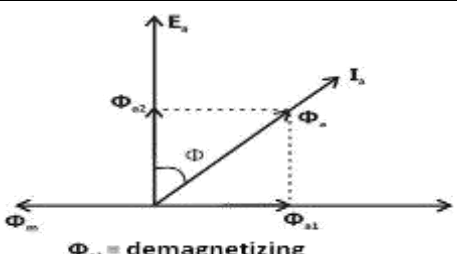
$$R = \frac{3}{2}R_m$$



This resistance is dc resistance but ac resistance is higher due to skin effect.

$$R_{a(ac)} = (1.2 \text{ to } 1.3)R$$

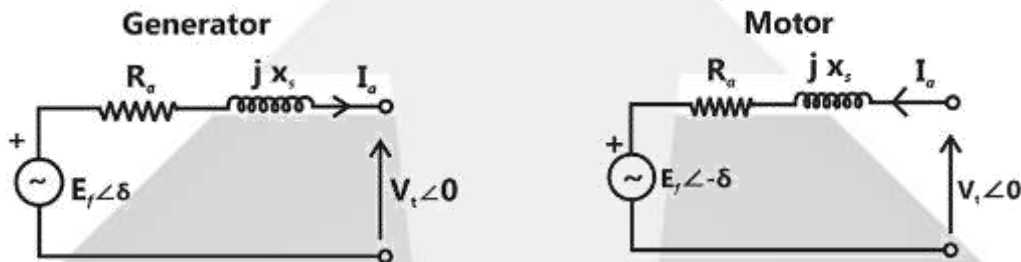
## Armature Reaction

Power factor	Generator	Motor
Unity	 <p>Purely cross magnetizing</p>	 <p>Purely cross magnetizing</p>
Zero pf lagging	 <p>Purely Demagnetizing</p>	 <p>Purely Magnetizing</p>
Zero pf leading	 <p>Purely Magnetizing</p>	 <p>Purely Demagnetizing</p>
Lagging pf ( $\cos\phi$ )	 <p><math>\Phi_{a1}</math> = demagnetizing <math>\Phi_{a2}</math> = cross magnetizing</p>	 <p><math>\Phi_{a1}</math> = magnetizing <math>\Phi_{a2}</math> = cross magnetizing</p>
Leading pf ( $\cos\phi$ )	 <p><math>\Phi_{a1}</math> = magnetizing <math>\Phi_{a2}</math> = cross magnetizing</p>	 <p><math>\Phi_{a1}</math> = demagnetizing <math>\Phi_{a2}</math> = cross magnetizing</p>

### Leakage Flux

Leakage flux links only one winding but not both so if it is present in stator, it won't link to rotor & vice versa.

### **Equivalent Circuit**



$X_s$  = synchronous reactance

$$= X_{ar} + X_l$$

= sum of armature reaction & leakage reactance

$$|E| \angle \delta = |V| \angle 0 + |I_a| \angle \phi (R_a + jX_s), \text{ for Synchronous Generator}$$

$$|E| \angle -\delta = |V| \angle 0 - |I_a| \angle \phi (R_a + jX_s), \text{ for Synchronous Motor}$$

Where  $\phi$  is power factor angle (leading)

for lagging power factor we replace  $\phi$  by " $-\phi$ "

### **Voltage Regulation**

$$\text{Voltage regulation} = \frac{|E| - |V|}{|V|} \times 100\%$$

### **For zero voltage regulation**

$$\theta + \phi = 180^\circ \quad \theta = \tan^{-1} \left( \frac{X_s}{R_a} \right)$$

$$\cos \phi = \text{load pf (leading)}$$

### For maximum voltage regulation

$$\theta = \phi$$

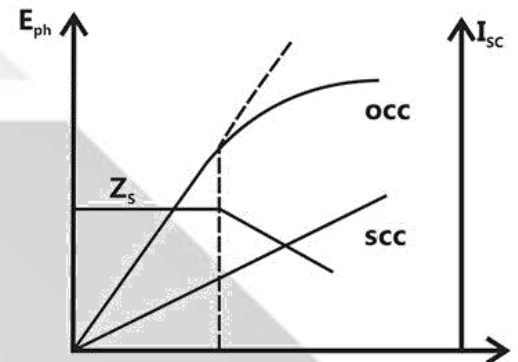
$$\cos \phi = \text{load pf (lagging)}$$

### Characteristics of Alternator

#### OCC & SCC

Open circuit characteristics & short circuit characteristics

$$Z_s = \frac{\text{open circuit voltage at same field current}}{\text{short circuit current at same field current}}$$



Generally, open circuit voltage is given as Line to Line value so, before calculating  $Z_s$ , we need to find phase voltage

$$Z_s = \frac{V_{oc}/\sqrt{3}}{I_{sc}} \Big|_{I_f = \text{constant}} \quad : \text{For Star Connection}$$

$$Z_s = \frac{V_{oc}}{I_{sc}} \Big|_{I_f = \text{constant}} \quad : \text{For Delta Connection}$$

#### Short circuit ratio

$$\text{SCR} = \frac{\text{Field current required for rated open circuit voltage}}{\text{Field current required for rated short circuit current}}$$

$$= \frac{1}{X_s (\text{pu})}$$

$X_s (\text{pu})$  = synchronous reactance in pu



## Finding Voltage Regulation

There are usually 4 methods to find voltage regulation

- EMF Method
- MMF Method
- Potier Triangle Method
- ASA Method

Order of voltage regulation: EMF > ASA > ZPF > MMF

## Power Angle Equation

### Output of generator

$$P_{out} = \frac{V_t E_f}{Z_s} \cos(\theta - \delta) - \frac{V_t^2}{Z_s} \cos \theta$$

$$Q_{out} = \frac{V_t E_f}{Z_s} \sin(\theta - \delta) - \frac{V_t^2}{Z_s} \sin \theta$$

### Input of motor

$$P_{in} = \frac{V_t^2}{Z_s} \cos \theta_s - \frac{V_t E_f}{Z_s} \cos(\theta + \delta)$$

$$Q_{in} = \frac{V_t^2}{Z_s} \sin \theta - \frac{V_t E_f}{Z_s} \sin(\theta + \delta)$$

Synchronous Impedance =  $Z_s = R_a + jX_s = |Z_s| \angle \theta$

$$\theta = \tan^{-1} \left( \frac{X_s}{R_a} \right)$$

If  $R_a$  = neglected,  $Z_s = jX_s = |X_s| \angle 90^\circ$

$$(P_{out})_g = \frac{E_f V_t}{X_s} \sin \delta; (Q_{out})_g = \frac{V_t}{X_s} (E_f \cos \delta - V_t)$$

### Developed power in synchronous motor

$$P_{dev} = \frac{E_f V_t}{Z_s} \cos(\theta - \delta) - \frac{E_f^2}{Z_s} \cos \theta$$

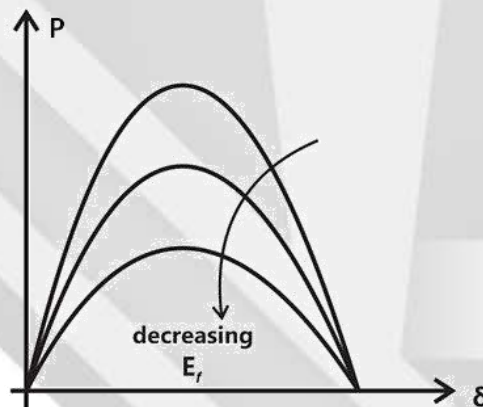
$$Q_{dev} = \frac{E_f V_t}{Z_s} \sin(\theta - \delta) - \frac{E_f^2}{Z_s} \sin \theta$$

If  $r_a$  is neglected,  $Z_s = X_s \angle 90^\circ$

$$P_{dev} = \frac{E_f V_t}{Z_s} \sin \delta$$

$$Q_{dev} = \frac{E_f V_t}{Z_s} \cos \delta - \frac{E_f^2}{Z_s}$$

- Developed Power is the power available at armature of motor.
- In all power expressions, all voltages are line voltages and if we want to use phase voltage, we must multiply all expressions by a factor of 3.

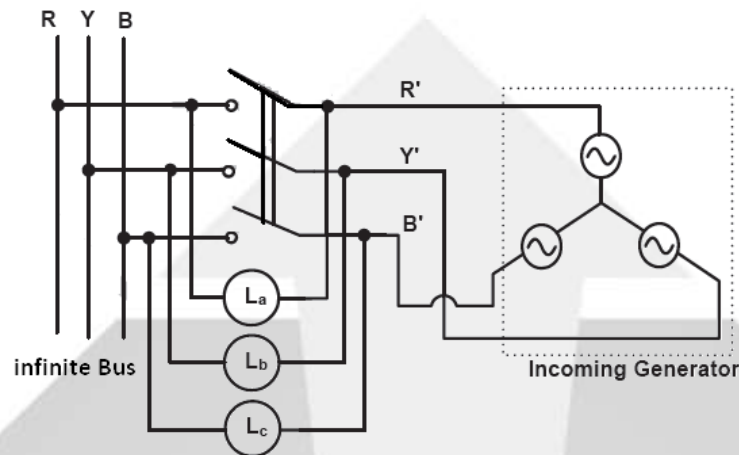


### Parallel operation of Alternators

Necessary Conditions

- 1) Terminal voltage of incoming alternator must be same as that of existing system.
- 2) Frequency should be same.
- 3) Phase sequence should be same.

## Synchronization by Lamp Method



- 1) Observe if 3 lamps are bright & dark simultaneously, that means phase sequence of incoming alternator is same as that of existing system.  
Otherwise, phase sequence is opposite and stator terminals must be interchanged to reverse phase sequence of incoming generator.

- 2) The frequency of alternator is usually a bit higher than infinite bus.

- 3) To understand the concept better, refer Ques. 39 of GATE – 2014 EE-01 paper.

- If two alternators are supplying a load and we change either excitation or steam input of one machine is varied, then following effects will happen:
- If excitation of machine 1 is increased

Parameter	Machine 1	Machine 2
Real Power	Same	Same
Reactive Power	Increases	Decreases
Armature Current	Increases	Decreases
Power Factor	Decreases	Increases

- If steam input of machine 1 is increased

Parameter	Machine 1	Machine 2
Real Power	Increases	Decreases
Reactive Power	Constant	Constant
Armature Current	Increases	Decreases
Power Factor	Increases	Decreases

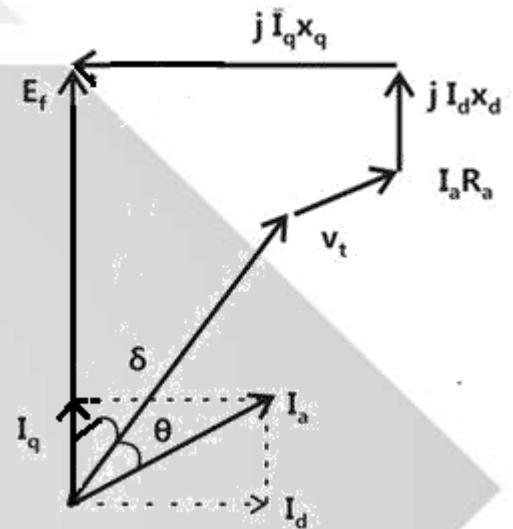
## Droop Characteristics

$$\text{droop of generator} = \frac{f_{NL} - f_{FL}}{f_{FL}} \times 100\%$$

Example: Refer Kuestions on Electrical Machines Type-8

## Salient Pole Machine

- In case of salient pole machine, There are 2 reactances  $X_d$  &  $X_q$   
 $X_d$  : Direct axis reactance  
 $X_q$  : quadrature axis reactance
- $I_d = I_a \sin(\delta + \theta) \angle \delta - 90$   
 $I_q = I_a \cos(\delta + \theta) \angle \delta$   
 $\psi = (\delta + \theta)$



### For synchronous generator

$$\tan \psi = \frac{V \sin \phi \pm I_a X_q}{V \cos \phi + I_a R_a} ; \begin{array}{l} + \rightarrow \text{lagging pf} \\ - \rightarrow \text{leading pf} \end{array}$$

### For synchronous motor

$$\tan \psi = \frac{V \sin \phi \pm I_a X_q}{V \cos \phi - I_a R_a} ; \begin{array}{l} + \rightarrow \text{leading pf} \\ - \rightarrow \text{lagging pf} \end{array}$$

## Power – Angle Characteristics

$$P = \underbrace{\frac{V_t E_f}{X_d} \sin \delta}_{\text{Excitation power}} + \underbrace{\frac{V_t^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta}_{\text{Reluctance power}}$$

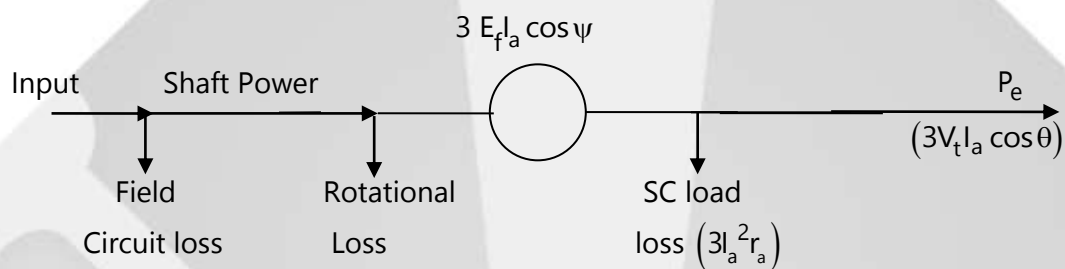
## Slip Test

If machine is run by prime mover at a speed other than synchronous speed & voltages & currents are observed

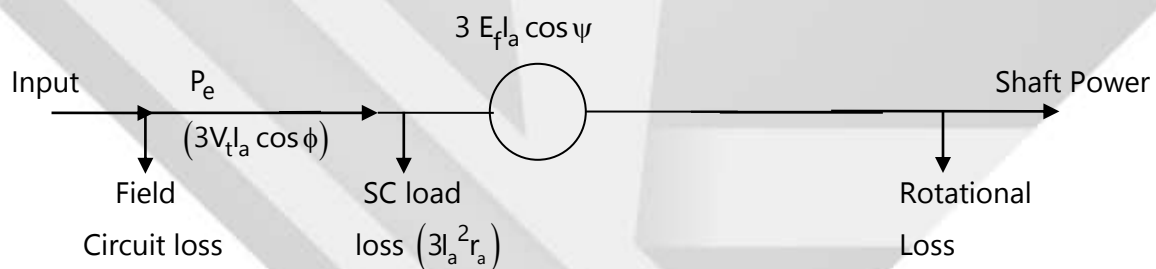
$$X_d = \frac{\text{Maximum Voltage}}{\text{Maximum Current}}$$

$$X_q = \frac{\text{Maximum Voltage}}{\text{Maximum Current}}$$

## Power Flow Diagram



Power Flow for Synchronous Generator



Power Flow Diagram for Synchronous Motor

## Induction Machines

### Stator & Rotor Magnetic Fields

- When a 3-phase supply is connected to the stator, then a magnetic field is set up whose speed of rotation is

$$N_s = \frac{120f}{p}$$

$f$  = frequency of supply

- If negative sequence currents are applied the rotating magnetic field rotates in opposite direction as compared to magnetic field produced by positive sequence currents.
- The rotor rotates in same direction as the stator magnetic field with a speed,  $N_r$ .

$$\text{slip } s = \frac{N_s - N_r}{N_s}$$

$$\therefore N_r = N_s(1 - s)$$

- Speed of rotor magnetic field with respect to rotor =  $sN_s$
  - speed of rotor magnetic field with respect to stator =  $N_s$ .
- Hence, stator & rotor magnetic fields are at rest with respect to each other.
- Frequency of emf & current in rotor =  $sf$

With respect to	Relative Speed of				
		Stator	Stator Magnetic Field	Rotor	Rotor Magnetic Field
	Stator	0	$N_s$	$N_s(1-s)$	$N_s$
	Stator Magnetic Field	$-N_s$	0	$-sN_s$	0
	Rotor	$-N_s(1-s)$	$sN_s$	0	$sN_s$
	Rotor Magnetic Field	$-N_s$	0	$-sN_s$	0

### Inverted Induction Motor

- When a 3- $\phi$  supply is connected to the rotor & stator terminals are shorted or are connected to the resistive load.
- Then a rotor magnetic field is set up which rotates at speed  $N_s$  with respect to rotor ;

$$N_s = \frac{120f}{p} \text{ where } f \text{ is frequency of supply.}$$

- If rotor rotates at speed  $N_r$ , then slip

$$s = \frac{N_s - N_r}{N_s}$$

Here, the rotor rotates in a direction opposite to the direction of rotation of stator magnetic field.

- Speed of rotor magnetic field with respect to stator

$$= N_s - N_s(1-s) = sN_s$$

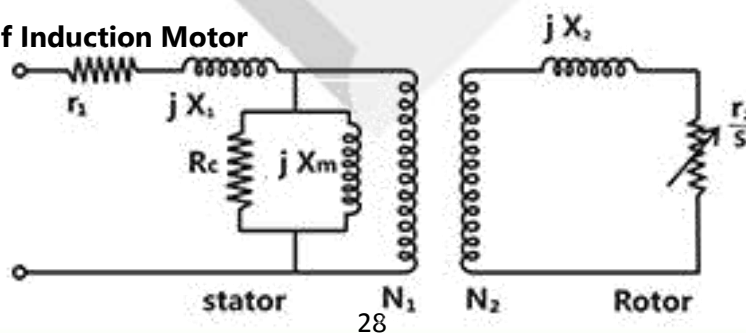
$$\text{Speed of stator magnetic field} = sN_s$$

- Frequency of emf & current induced in stator =  $sf$

$f$  = supply frequency on rotor.

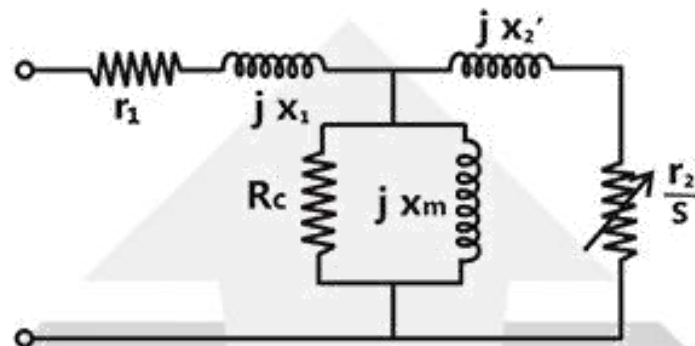
With respect to	Relative Speed of				
		Stator	Stator Magnetic Field	Rotor	Rotor Magnetic Field
	Stator	0	$sN_s$	$N_s(1-s)$	$sN_s$
	Stator Magnetic Field	$-sN_s$	0	$-N_s$	0
	Rotor	$-N_s(1-s)$	$N_s$	0	$N_s$
	Rotor Magnetic Field	$-sN_s$	0	$-N_s$	0

### Equivalent circuit of Induction Motor





If we refer all parameters on stator side



$$r_2' = r_2 \left( \frac{N_1'}{N_2'} \right)^2 ; x_2' = x_2 \left( \frac{N_1'}{N_2'} \right)^2$$

$$N_1' = N_1 k\omega_1$$

Where  $N_1$  = no. of turns per phase on stator

$k\omega_1$  = winding factor of stator winding

$$N_2' = N_2 k\omega_2$$

$N_2$  = number of turns per phase on rotor

$k\omega_2$  = winding factor of rotor winding

### Tests Conducted on Induction Motor

#### (i) No-Load Test

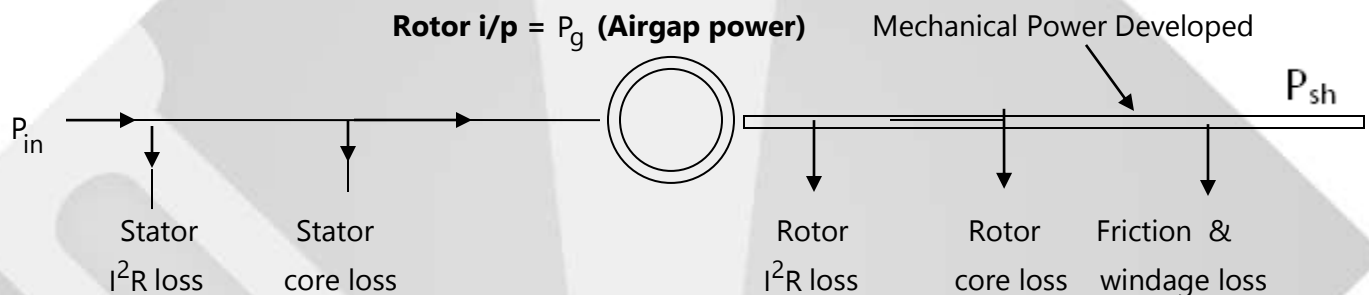
- Conducted on Stator with no-load on rotor side.
- It gives No-Load Losses ( Rotational Loss + Core Loss).

#### (ii) Blocked Rotor Test

- Conducted on stator side keeping rotor blocked
- It gives full load Copper Losses and equivalent resistance and equivalent reactance referred to Stator Side.

- $R_{01}$  &  $X_{01}$  are equivalent winding resistance & equivalent leakage reactor referred to Stator side.
- Wattmeter reading =  $P = I_{sc}^2 R_{01}$  from this equation, we can calculate  $R_{01}$
- $Z_{01} = \frac{V_{sc}}{I_{sc}}$  &  $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$
- We obtain  $R_{01}$ ,  $X_{01}$  & full load copper losses from this test.
- $R_{01} = R_1 + R_2'$ ;  $X_{01} = X_1 + X_2'$

### Power Flow Diagram



$$P_g = \frac{3I_2^2 r_2}{s}$$

$I_2$  = rotor current

$s$  = slip

$r_2$  = rotor resistance per phase

$$\text{Rotor } C_u \text{ Loss} = 3I_2^2 r_2 = sP_g$$

$$\text{Mechanical power developed} = P_g - sP_g = (1-s)P_g$$

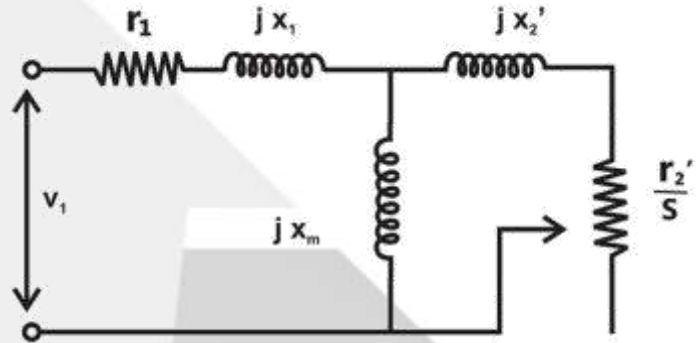
$$\text{Developed Torque, } T_e = \frac{P_m}{\omega_r} = \frac{(1-s)P_g}{(1-s)\omega_s} = \frac{P_g}{\omega_s}$$

## Torque – Slip Characteristics

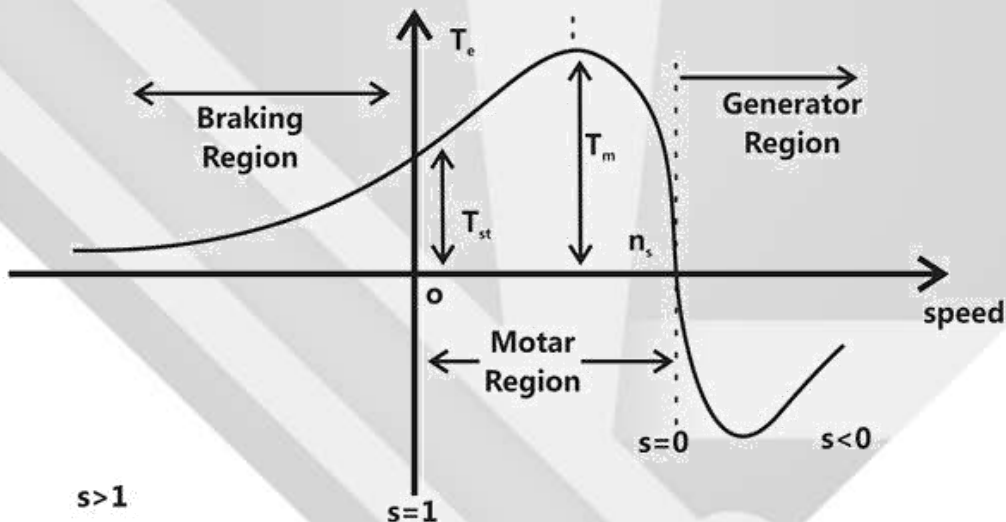
If core loss is neglected then equivalent circuit looks like as shown

$$\bar{V}_e = \frac{\bar{V}_1 (jX_m)}{r_1 + j(X_1 + X_m)}$$

$$R_e = \frac{r_1 X_m}{X_1 + X_m} ; X_e = \frac{X_1 X_m}{(X_1 + X_m)}$$



$$\text{Torque developed, } T_c = \frac{m V_e^2}{w_s \left[ \left( R_e + \frac{r_2'}{s} \right)^2 + (X_2' + X_e)^2 \right]} \times \frac{r_2'}{s}$$



For Approximate analysis,

$$\text{Stator impedance is neglected; } T_c = \frac{3}{w_s} \frac{V_1^2}{\left\{ \left( \frac{R_2'}{s} \right)^2 + X_2'^2 \right\}} \times \frac{r_2'}{s}$$

- At low slip,  $s \ll 1$

$$\frac{R_2'}{s} \gg X_2', \quad T_c = \frac{3}{\omega_s} \times \frac{sV_1^2}{R_2'} \Rightarrow T_c \propto s$$

- At high slip,  $s \approx 1$

$$\frac{R_2'}{s} \ll X_2', \quad T_c = \frac{3}{\omega_s} \left( \frac{V_1}{X_2'} \right)^2 \frac{R_2'}{s} \propto \frac{1}{s}$$

### For maximum torque

$$S_{m,T} = \frac{R_2'}{\sqrt{R_e^2 + (X_e + X_2')^2}}$$

It stator impedance is neglected

$$S_{m,T} = \frac{R_2'}{X_2'} \text{ and } T_{\max} = \frac{3}{\omega_s} \frac{V_1^2}{(2X_2')}$$

And also,  $\frac{T}{T_{\max}} = \frac{2}{\left( \frac{s}{S_{m,T}} + \frac{S_{m,T}}{s} \right)}$ , where T is the torque at a slip 's'

### For maximum power

$$S_{m,P} = \frac{R_2'}{\sqrt{(R_e + R_2')^2 + (X_e + X_2')^2} + R_2'}$$

### Starting of Induction Motor

#### (i) Direct on – line starting

- Directly motor is connected to supply.

$$\frac{T_{e,st}}{T_{e,FL}} = \left( \frac{I_{st}}{I_{FL}} \right)^2 S_{FL}$$

### (ii) Auto Transformer Starting

- Instead of connecting the motor to direct supply we reduce the voltage from  $V_1$  to  $(xV_1)$
- This is done with the help of auto – transformer.
- $\frac{T_{e,st}}{T_{e,FL}} = \frac{1}{X^2} \left( \frac{I_{st}}{I_{FL}} \right)^2 S_{FL}$
- $\frac{T_{e,st} \text{ (auto X'mer)}}{T_{e,FL} \text{ (direct)}} = \left( \frac{XV_1}{V_1} \right)^2 = X^2$

### (iii) Star – Delta Starting

- At starting, stator winding is connected in star & in running state stator winding is connected in delta.
- $V_{ph} = \frac{V_1}{\sqrt{3}} ; \quad \frac{T_Y}{T_D} = \frac{\left( \frac{V_1}{\sqrt{3}} \right)^2}{V_1^2} = \frac{1}{3}$
- $I_Y = \frac{1}{\sqrt{3}} I_D$
- $\frac{T_{st}}{T_{FL}} = \left( \frac{I_{st,Y}}{I_{FL,d}} \right)^2 S_{FL} = \left( \frac{\frac{1}{\sqrt{3}} I_{st,d}}{I_{FL,d}} \right)^2 S_{FL} ; \quad \frac{T_{st}}{T_{FL}} = \frac{1}{3} \left( \frac{I_{st,Y}}{I_{FL,d}} \right)^2 S_{FL}$

### Speed Control of Induction Motor

- Constant  $V/f$  Control

At low slip,  $T = \frac{180}{2\pi N_s} \cdot \frac{sV_1^2}{R_2'}$

$$s = \frac{(N_s - N)}{N_s}$$

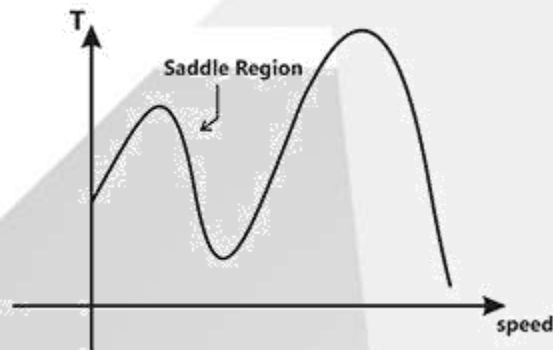
$$T = \frac{180}{2\pi N_s} \cdot \frac{(N_s - N)}{N_s} \cdot \frac{V_1^2}{R_2'} \propto \left( \frac{V_1}{f} \right)^2 (N_s - N)$$

For constant torque,  $(N_s - N) = \text{constant}$

So, by varying frequency we vary  $N_s$  & since  $(N_s - N) = \text{constant}$  we vary  $N$  accordingly.

### Crawling

- Due to harmonics, the actual torque characteristics may look like



- Due to this saddle region, the motor may become stable at a low speed & this is called as crawling.

### Cogging

- If number of stator slots is equal to or integral multiple number of rotor slots, then at the time of start, the strong alignment forces between stator teeth & rotor teeth simultaneously at all rotor teeth may prevent movement of rotor. This is called cogging.

## Single Phase Induction Motor

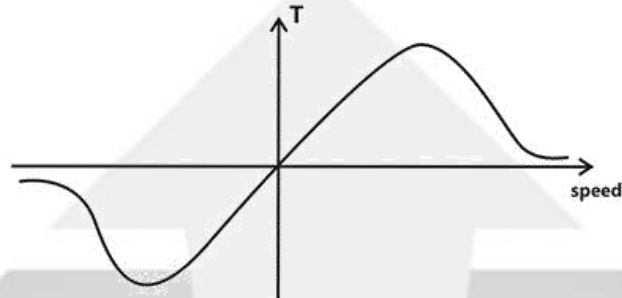
- According to Double field Revolving Theory, a single phase mmf can be resolved into two rotating fields one rotating clockwise called as Forward field & other rotating anti-clock wise called as Backward Field.

Both fields rotate at synchronous speed

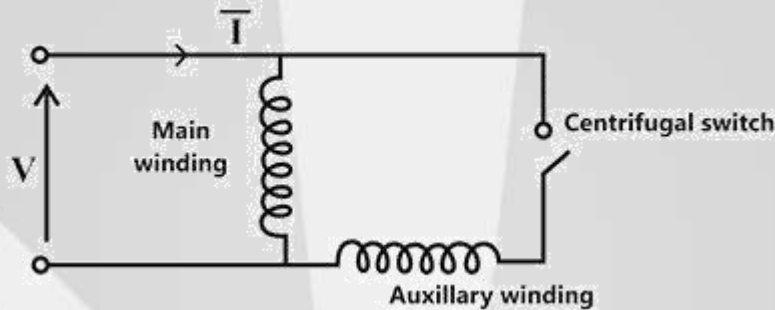
$$N_s = \frac{120f}{P}$$

- If rotor rotates at speed  $N_r$ , or a slips with respect to forward field.  
Then slip with respect to backward field is  $(2 - s)$

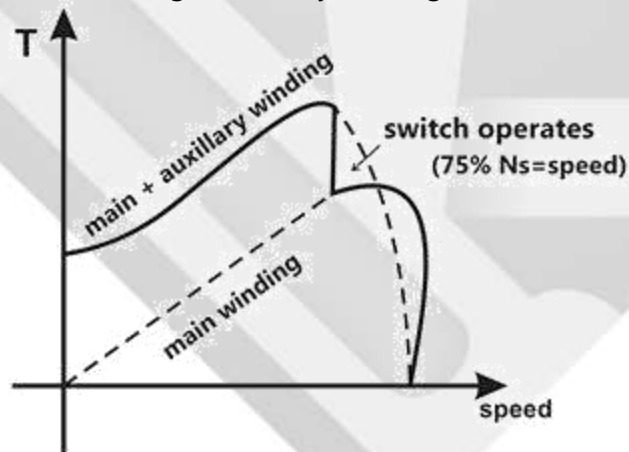
- Due to these two fields producing opposing torques on rotor single phase IM is not self starting.



- To produce starting torque, we introduce an auxiliary winding which is used at the time of start & is disconnected during the run stage.

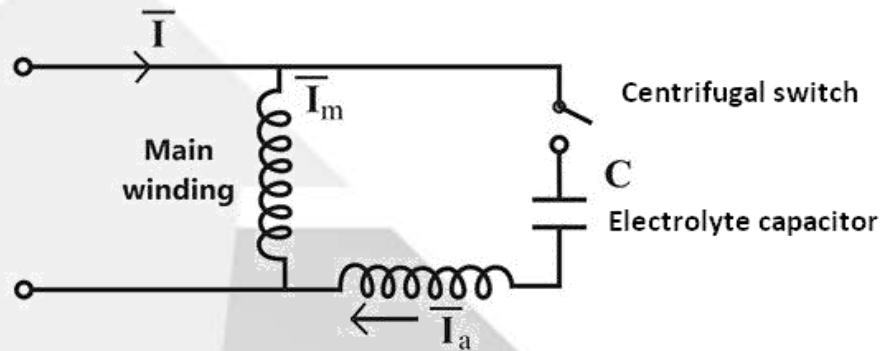
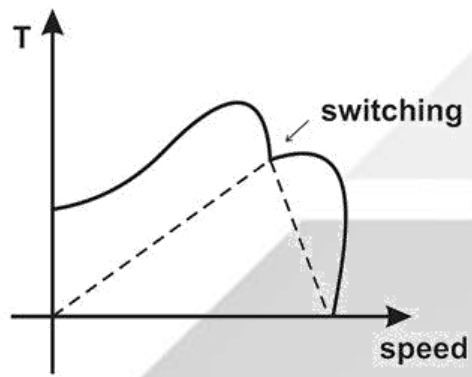


We generally design auxiliary winding such that phase difference is approximately  $90^\circ$  between main winding & auxiliary winding currents.

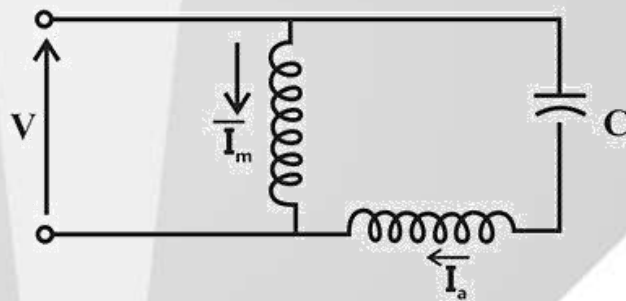
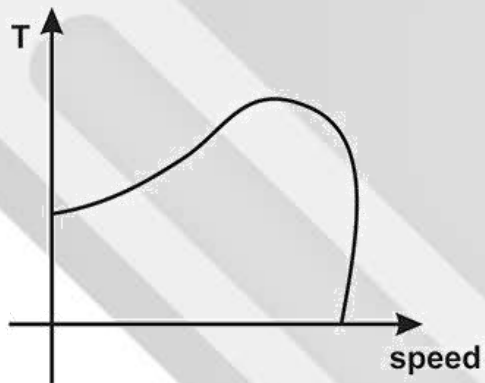




### ○ Capacitor Start Motor



### ○ Capacitor Run Motor



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**[info@kreatryx.com](mailto:info@kreatryx.com)**

**[kreatryx.thegateguru@gmail.com](mailto:kreatryx.thegateguru@gmail.com)**

**+91 7406144144**

**+91 9819373645**

**0120-4326333**

**Address - SE 617, Shastri Nagar, Ghaziabad, U.P (201002)**

**[www.kreatryx.com](http://www.kreatryx.com)**